NORMANHURST BOYS HIGH SCHOOL

MATHEMATICS EXTENSION 1

2022 Year 12 Course Assessment Task 4 (Trial Examination)
Tuesday August 16, 2022

General instructions

- Working time – 2 hours.
  (plus 10 minutes reading time)
- Write using blue or black pen. Where diagrams are to be sketched, these may be done in pencil.
- NESA approved calculators may be used.
- Attempt **all** questions.
- At the conclusion of the examination, bundle the booklets used in the correct order within this paper and hand to examination supervisors.

SECTION I

- Mark your answers on the answer grid provided (on page 9)

SECTION II

- Commence each new question on a new booklet. Write on both sides of the paper.
- All necessary working should be shown in every question. Marks may be deducted for illegible or incomplete working.

NESA STUDENT #: ..... **# BOOKLETS USED**: .....

Class (please ✔)

- ◯ 12MAX.1 – Mr Lam
- ◯ 12MAX.2 – Ms C. Kim
- ◯ 12MAX.3 – Ms Lee
- ◯ 12MXX.1 – Ms Ham
- ◯ 12MXX.2 – Mr Sekaran

Marker's use only.

QUESTION	1-10	11	12	13	14	Total	%
MARKS	/10	/18	/14	/13	/15	/70	

Section I

10 marks
Attempt Question 1 to 10
Allow approximately 15 minutes for this section

Mark your answers on the answer grid provided (labelled as page 9).

Questions **Marks**

1. Which of the following is the next step in progressing to the correct solution to the following inequality? **1**

$$\frac{x^2-4}{x} \geq 0$$

(A) $x^2-4 \geq 0$

(B) $x(x^2-4) \geq 0$

(C) $x^2-4 \leq 0$

(D) $x(x^2-4) \leq 0$

2. Given $\tan\theta = \dfrac{1}{3}$, what is the exact value of **1**

$$\tan\left(\theta + \frac{\pi}{3}\right)$$

(A) $\dfrac{\sqrt{3}+3}{3\sqrt{3}-1}$

(B) $\dfrac{\sqrt{3}-3}{3\sqrt{3}+1}$

(C) $\dfrac{1+3\sqrt{3}}{3-\sqrt{3}}$

(D) $\dfrac{1-3\sqrt{3}}{3+\sqrt{3}}$

3. Which Cartesian equation is equivalent to the parametric equations $x = 5\cos\theta$ and $y = 6\sin\theta$? **1**

(A) $\dfrac{x}{5} + \dfrac{y}{6} = 1$

(B) $\dfrac{x}{25} + \dfrac{y}{36} = 1$

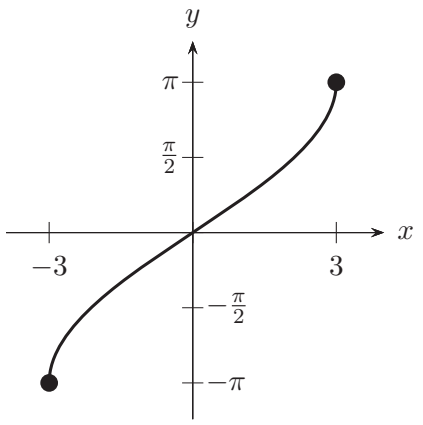
(C) $\dfrac{x^2}{25} + \dfrac{y^2}{36} = 1$

(D) $x^2 + y^2 = 61$

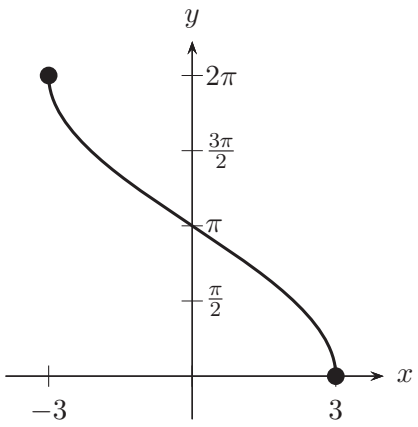
Examination continues overleaf...

4. Which graph best represents $y = 2\sin^{-1}\dfrac{x}{3}$? **1**

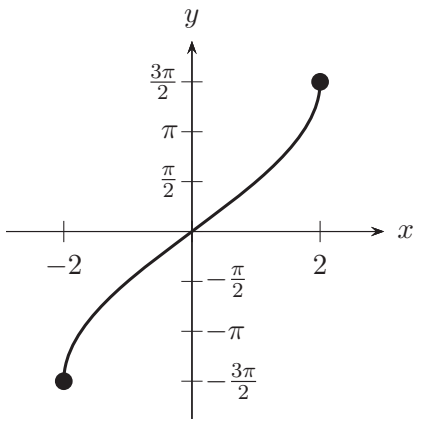
(A)



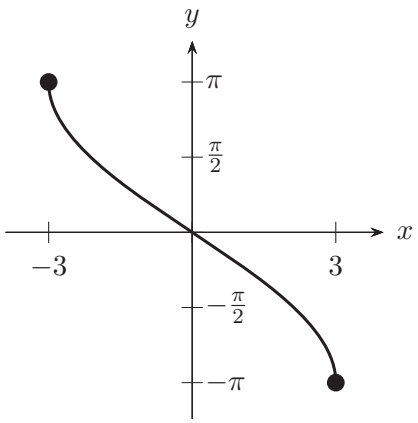
(C)



(B)



(D)



5. The equation $x^3 - 2x^2 + 4x - 3 = 0$ has roots $x = \alpha$, $x = \beta$ and $x = \gamma$. **1**
Find the value of $\dfrac{1}{\alpha} + \dfrac{1}{\beta} + \dfrac{1}{\gamma}$.

(A) $\dfrac{4}{3}$

(B) $-\dfrac{4}{3}$

(C) $\dfrac{3}{4}$

(D) $-\dfrac{3}{4}$

6. Let $\underset{\sim}{a}$ and $\underset{\sim}{b}$ be two non-zero vectors which are neither parallel nor perpendicular such that $\underset{\sim}{c} = \text{proj}_{\underset{\sim}{b}}\,\underset{\sim}{a}$. Which statement is always TRUE? **1**

(A) $(\underset{\sim}{c} + \underset{\sim}{a}) \cdot \underset{\sim}{b} = 0$

(B) $(\underset{\sim}{a} - \underset{\sim}{b}) \cdot \underset{\sim}{c} = 0$

(C) $(\underset{\sim}{a} + \underset{\sim}{b}) \cdot \underset{\sim}{c} = 0$

(D) $(\underset{\sim}{c} - \underset{\sim}{a}) \cdot \underset{\sim}{b} = 0$

Examination continues overleaf...

7. Which of the following is the derivative of $f(x) = (\sin x)(\sin^{-1} x)$? **1**

(A) $f'(x) = -\cos x \sin^{-1} x + \dfrac{\sin x}{\sqrt{1-x^2}}$

(B) $f'(x) = \cos x \sin^{-1} x + \dfrac{\sin x}{\sqrt{1-x^2}}$

(C) $f'(x) = -\sin x \sin^{-1} x + \dfrac{\cos x}{\sqrt{1-x^2}}$

(D) $f'(x) = \cos x \sin^{-1} x - \dfrac{\sin x}{\sqrt{1-x^2}}$

8. What is the primitive of $\dfrac{2f'(x)}{1+4(f(x)^2)}$? **1**

(A) $\tan^{-1}(f(2x))$

(B) $2\tan^{-1}(f(x))$

(C) $2\ln 2(f(x))$

(D) $\tan^{-1}(2f(x))$

9. The expression $3\cos x + 2\sin x$ is written in the form $R\cos(x-\theta)$, where $R > 0$ and $0 \leq \theta \leq \dfrac{\pi}{2}$. **1**

What is the value of $\tan\theta$?

(A) $-\dfrac{3}{2}$

(B) $-\dfrac{2}{3}$

(C) $\dfrac{2}{3}$

(D) $\dfrac{3}{2}$

10. What is the area enclosed between the curve $y = x^3 - 1$, the y-axis and the lines $y = 1$ and $y = 2$? **1**

(A) $\displaystyle\int_1^2 (x^3 - 1)\, dy$

(B) $\displaystyle\int_1^2 \sqrt[3]{y+1}\, dy$

(C) $\displaystyle\int_1^2 \left(\sqrt{3}y + 1\right)\, dy$

(D) $\displaystyle\int_1^2 (y+1)\, dy$

Examination continues overleaf...

Section II

60 marks
Attempt Questions 11 to 14
Allow approximately 1 hour and 45 minutes for this section.

Write your answers in the writing booklets supplied. Additional writing booklets are available. Your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (18 Marks) Commence a NEW booklet. **Marks**

(a) i. Show that **1**

$$\frac{1}{(n+1)!} - \frac{n+1}{(n+2)!} = \frac{1}{(n+2)!}$$

ii. Use mathematical induction to prove that, for all integers $n \geq 1$, **3**

$$\frac{1}{2!} + \frac{2}{3!} + \frac{3}{4!} + \cdots + \frac{n}{(n+1)!} = 1 - \frac{1}{(n+1)!}$$

(b) Find the constant term in the expansion of $\left(3x - \dfrac{1}{x^2}\right)^6$. **3**

You may leave your answers in the form $\binom{n}{k} a^b$ where $n, k, a, b \in \mathbb{Z}^+$.

(c) A school has twenty prefects. Jun and Alex are two of those prefects.
A committee of seven prefects is to be formed to organise a function.

i. How many different committees can be formed? **1**

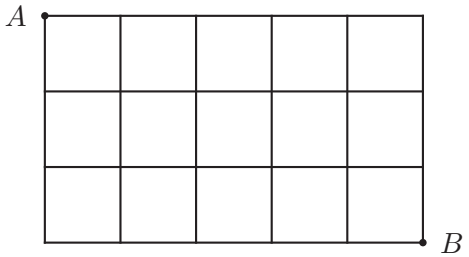
ii. What is the probability that the committee contains Jun but does not contain Alex? **2**

(d) A eleven-member Fund Raising Committee consists of five students, four teachers and two parents. The committee meets around a circular table.

i. How many different arrangements of the eleven members around the table are possible if the students sit together as a group and so do the teachers, but no teacher sits next to a student? **3**

ii. One of the students is the child of one of the parents present. Given that all arrangements in **(i)** are equally likely, what is the probability that the student sits next to their parent? **3**

(e) The diagram shows a grid consisting of unit squares. Chris needs to travel from point A to point B but can only do so by moving right or down along the grid lines. **2**

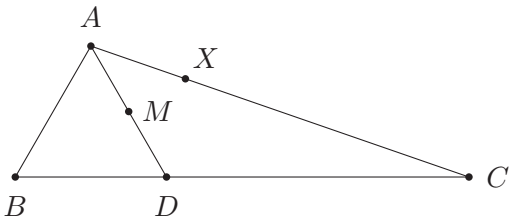


How many paths are there for Chris?

Examination continues overleaf...

Question 12 (14 Marks) Commence a NEW booklet. **Marks**

(a) The diagram below shows $\triangle ABC$ whose vertices have position vectors $\underset{\sim}{\text{a}}$, $\underset{\sim}{\text{b}}$ and $\underset{\sim}{\text{c}}$ from an origin O.



Point D lies on BC such that $BD = \dfrac{1}{3}BC$, point X lies AC such that $AX = \dfrac{1}{4}AC$ and M is the midpoint of AD.

i. Show that $\overrightarrow{AM} = -\dfrac{1}{2}\underset{\sim}{\text{a}} + \dfrac{1}{3}\underset{\sim}{\text{b}} + \dfrac{1}{6}\underset{\sim}{\text{c}}$. **3**

ii. Show that B, M and X are collinear. **2**

iii. Hence find $BM : MX$. **1**

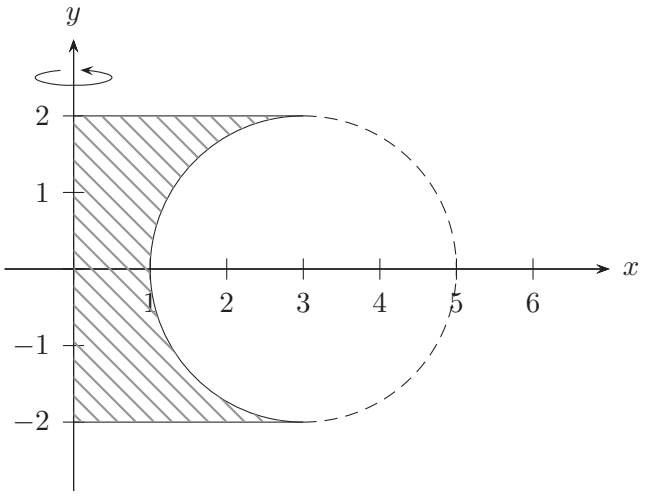
(b) i. Show that **1**

$$\frac{x^2}{x+1} = x - 1 + \frac{1}{x+1}$$

ii. Use the substitution $u = \sqrt{x-1}$ to show that **3**

$$\int_2^5 \frac{x-1}{x-1+\sqrt{x-1}}\,dx = 2\left(\frac{1}{2} + \ln\frac{3}{2}\right)$$

(c) To make the rim, the region between the circle $(x-3)^2 + y^2 = 4$, the lines $y = -2$, $y = 2$ and the y-axis is rotated about the y axis. **4**



Find the exact volume of the steel needed to make the rim.

Examination continues overleaf...

Question 13 (13 Marks) Commence a NEW booklet. **Marks**

(a) On the grid supplied on page 10 and addition of ordinates, sketch the graph of **3**

$$y = x - 2\sin x \text{ for } 0 \leq x \leq 2\pi$$

Show all important features.

(b) Find $\int \sin^2 3x \, dx$. **2**

(c) i. You are given that $\cos 3\theta = 4\cos^3\theta - 3\cos\theta$ and $\sin 3\theta = 3\sin\theta - 4\sin^3\theta$. **3**
Use this to show that

$$\sin^4\theta + \cos^4\theta = \frac{1}{4}(\cos 4\theta + 3)$$

ii. By letting $x = \cos\theta$, show that the equation below has roots **3**
$\pm\cos\dfrac{\pi}{12}$ and $\pm\cos\dfrac{5\pi}{12}$.

$$8x^4 + 8\left(1 - x^2\right)^2 = 7$$

iii. Deduce that **2**

$$\cos\frac{\pi}{12}\cos\frac{5\pi}{12} = \frac{1}{4}$$

Examination continues overleaf...

Question 14 (15 Marks) Commence a NEW booklet. **Marks**

(a) The velocity of a particle satisfies the differential equation $\dfrac{dx}{dt} = x\sin(t)$, where x is its displacement in centimetres relative to a fixed point O at time t seconds.

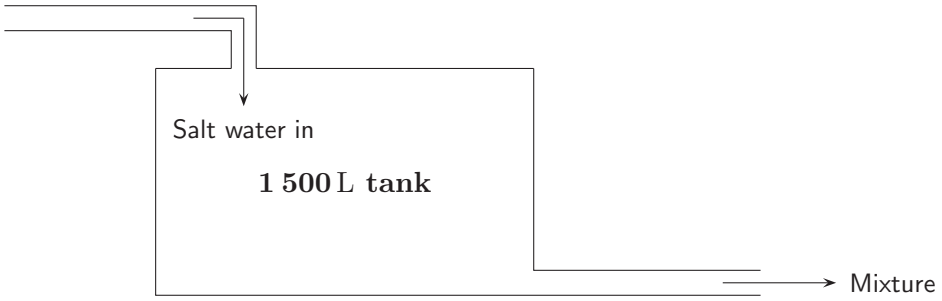
Initially, the displacement of the particle is 1 cm.

i. Find an expression for x in terms of t. **3**

ii. Find when the particle first reaches its maximum displacement within $0 \leq t < 2\pi$. **2**

(b) In the diagram, a tank initially contains 1500 litres of pure water.

Salty water begins pouring into the tank from a pipe, and is stirred to ensure that it is always completely mixed with the pure water.



A second pipe draws the mixture off at the same rate, so that there is always a total of 1500 litres in the tank.

i. If the salty water entering the tank contains 3 grams of salt per litre, and is flowing in at the constant rate of w litres/min, how much salt is entering the tank per minute? **1**

ii. If there are Q grams of salt in the tank at time t, how much salt is in 1 litre at time t? **1**

iii. Hence write down the amount of salt leaving the tank per minute. **2**

iv. Use the previous parts to show that $\dfrac{dQ}{dt} = \dfrac{w}{1500}(4500 - Q)$. **2**

v. By solving the differential equation in part (iv), show that **3**

$$Q = 4500 + Ae^{-\frac{wt}{1500}}$$

vi. Determine the value of A. **1**

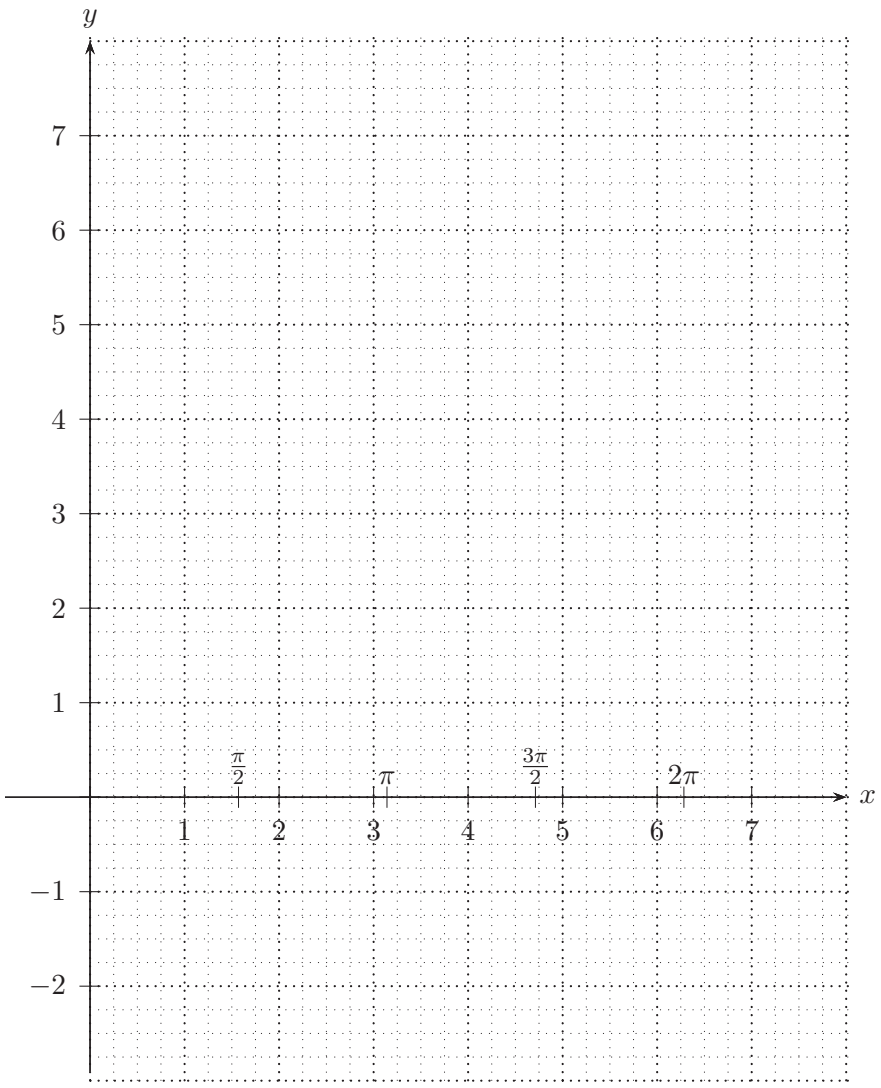
End of paper.

Question 13

(a) Using the grid supplied below and addition of ordinates, sketch the graph of **3**

$$y = x - 2\sin x \text{ for } 0 \leq x \leq 2\pi$$

Show all important features.



MC

1. B 2. C 3. C 4. A 5. A 6. D 7. B 8. D 9. C 10. B

Short Answer

Question 11. (Miss C. Kim)

a) i) $\text{LHS} = \frac{1}{(n+1)!} - \frac{n+1}{(n+2)!}$

$= \frac{n+2}{(n+2)(n+1)!} - \frac{n+1}{(n+2)!}$

$= \frac{n+2-(n+1)}{(n+2)!}$

$= \frac{1}{(n+2)!}$

$= \text{RHS}$

ii) Let $P(n)$ be given proposition.

Base case : Prove true for $n=1$

$\text{LHS} = \frac{1}{(1+1)!} = \frac{1}{2}$ $\quad \text{RHS} = 1 - \frac{1}{(1+1)!} = \frac{1}{2}$

$\therefore \text{LHS} = \text{RHS}$

$\therefore P(1)$ is true.

Assume true for $n=k$;

$\frac{1}{2!} + \frac{2}{3!} + \frac{3}{4!} + \cdots + \frac{k}{(k+1)!} = 1 - \frac{1}{(k+1)!}$

Prove true for $n=k+1$;

RTP: $\frac{1}{2!} + \frac{2}{3!} + \frac{3}{4!} + \cdots + \frac{k}{(k+1)!} + \frac{k+1}{(k+2)!} = 1 - \frac{1}{(k+2)!}$

$\text{LHS} = 1 - \frac{1}{(k+1)!} + \frac{k+1}{(k+2)!}$ (By assumption)

$= 1 - \left(\frac{1}{(k+1)!} - \frac{k+1}{(k+2)!}\right)$

$= 1 - \frac{1}{(k+2)!}$ [from i)]

$= \text{RHS}$

$\therefore P(k+1)$ is true for all $n \in \mathbb{Z}^+$

$\therefore$ **True by Mathematical Induction.**

b) $\left(3x - \frac{1}{x^2}\right)^6 = \sum_{k=0}^{6} \binom{6}{k} (3x)^{6-k} (-x^{-2})^k$

constant term when $x^{6-k} \times x^{-2k} = 1$

$6 - 3k = 0$

$3k = 6$

$k = 2$

$\therefore$ Constant term $= \binom{6}{2} 3^4 (-1)^2$

$= \binom{6}{2} 3^4$

$= 1215$

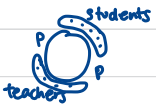
c) i) ${}^{20}C_7 = 77520$

ii) ${}^{18}C_6 = 18\,564$

$\therefore P(\text{Jun but not Alex}) = \frac{{}^{18}C_6}{{}^{20}C_7}$

$= \frac{91}{380}$

d) 5 Students, 4 teachers, 2 parents

i) 

(1) (1) (1)

$5! \times 4! \times 2$

$= 5760$

ii) $4! \times 4! \times 2 = 1152$

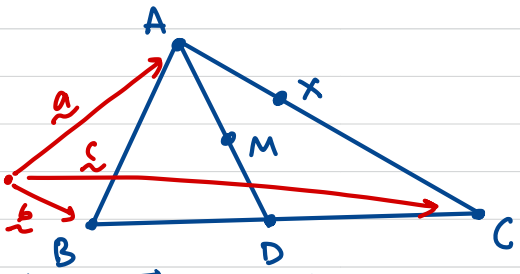
$\therefore P(\text{student sits next to their parent}) = \frac{1152}{5760}$

$= \frac{1}{5}$

e) $\frac{8!}{5!3!} = 56$ (1) (1) ${}^8C_3$ or ${}^8C_5$.

Question 12 (Mr Lam)

a)



i) $\overrightarrow{AB} = \underset{\sim}{b} - \underset{\sim}{a}$, $\overrightarrow{AC} = \underset{\sim}{c} - \underset{\sim}{a}$, $\overrightarrow{BC} = \underset{\sim}{c} - \underset{\sim}{b}$

$\overrightarrow{BD} = \frac{1}{3}\overrightarrow{BC}$

$= \frac{1}{3}(\underset{\sim}{c} - \underset{\sim}{b})$

$\overrightarrow{AD} = \overrightarrow{AB} + \overrightarrow{BD}$

$= \underset{\sim}{b} - \underset{\sim}{a} + \frac{1}{3}(\underset{\sim}{c} - \underset{\sim}{b})$

$= -\underset{\sim}{a} + \frac{2}{3}\underset{\sim}{b} + \frac{1}{3}\underset{\sim}{c}$

$\overrightarrow{AM} = \frac{1}{2}\overrightarrow{AD}$

$= \frac{1}{2}\left(-\underset{\sim}{a} + \frac{2}{3}\underset{\sim}{b} + \frac{1}{3}\underset{\sim}{c}\right)$

$= -\frac{1}{2}\underset{\sim}{a} + \frac{1}{3}\underset{\sim}{b} + \frac{1}{6}\underset{\sim}{c}$

ii) $\overrightarrow{AX} = \frac{1}{4}\overrightarrow{AC}$

$= \frac{1}{4}(\underset{\sim}{c} - \underset{\sim}{a})$

$= \frac{1}{4}\underset{\sim}{c} - \frac{1}{4}\underset{\sim}{a}$

$\overrightarrow{BM} = \overrightarrow{BA} + \overrightarrow{AM}$

$= -\underset{\sim}{b} + \underset{\sim}{a} + \left(-\frac{1}{2}\underset{\sim}{a} + \frac{1}{3}\underset{\sim}{b} + \frac{1}{6}\underset{\sim}{c}\right)$

$= \frac{1}{2}\underset{\sim}{a} - \frac{2}{3}\underset{\sim}{b} + \frac{1}{6}\underset{\sim}{c}$

$\overrightarrow{MX} = \overrightarrow{MA} + \overrightarrow{AX}$

$= -\left(-\frac{1}{2}\underset{\sim}{a} + \frac{1}{3}\underset{\sim}{b} + \frac{1}{6}\underset{\sim}{c}\right) + \frac{1}{4}\underset{\sim}{c} - \frac{1}{4}\underset{\sim}{a}$

$= \frac{1}{4}\underset{\sim}{a} - \frac{1}{3}\underset{\sim}{b} + \frac{1}{12}\underset{\sim}{c}$

$= \frac{1}{2}\left(\frac{1}{2}\underset{\sim}{a} - \frac{2}{3}\underset{\sim}{b} + \frac{1}{6}\underset{\sim}{c}\right)$

$= \frac{1}{2}\cdot\overrightarrow{BM}$

$\therefore \overrightarrow{MX} \parallel \overrightarrow{BM}$

$\therefore$ B, M and X are collinear.

iii) from ii) $\overrightarrow{MX} = \frac{1}{2}\overrightarrow{BM}$

$\therefore BM : MX = 2 : 1$

b) i) $RHS = x - 1 + \frac{1}{x+1}$

$= \frac{x(x+1) - (x+1) + 1}{x+1}$

$= \frac{x^2 + x - x - 1 + 1}{x+1}$

$= \frac{x^2}{x+1}$

$= LHS$

$\therefore \frac{x^2}{x+1} = x - 1 + \frac{1}{x+1}$

ii) $u = (x-1)^{\frac{1}{2}}$ when $x = 5, u = 2$; $x = 2, u = 1$

$du = \frac{1}{2}(x-1)^{-\frac{1}{2}}\,dx$

$= \frac{1}{2\sqrt{x-1}}\,dx$

$\therefore 2u\,du = dx$

$\int_2^5 \frac{x-1}{x-1+\sqrt{x-1}}\,dx = \int_1^2 \frac{u^2}{u^2+u}\,2u\,du$

$= 2\int_1^2 \frac{u^2}{u+1}\,du$

$= 2\int_1^2 u - 1 + \frac{1}{u+1}\,du$ (from i))

$= 2\left[\frac{u^2}{2} - u + \ln(u+1)\right]_1^2$

$= 2\left[\frac{2^2}{2} - 2 + \ln(2+1) - \left(\frac{1^2}{2} - 1 + \ln(1+1)\right)\right]$

$= 2\left(\ln 3 + \frac{1}{2} - \ln 2\right)$

$= 2\left(\ln\frac{3}{2} + \frac{1}{2}\right)$

c) $V = \pi \int_a^b x^2 \, dy$

$(x-3)^2 + y^2 = 4$

$(x-3)^2 = 4 - y^2$

$x - 3 = \pm\sqrt{4-y^2}$

$x = 3 \pm \sqrt{4-y^2}$

BUT LHS of semi-circle

$\therefore x = 3 - \sqrt{4-y^2}$

$x^2 = (3 - \sqrt{4-y^2})^2$

$= 9 - 6\sqrt{4-y^2} + 4 - y^2$

$= 13 - 6\sqrt{4-y^2} - y^2$

$\therefore V = \pi \int_{-2}^{2} 13 - 6\sqrt{4-y^2} - y^2 \, dy$

$= 2\pi \int_0^2 13 - 6\sqrt{4-y^2} - y^2 \, dy$

$= 2\pi \int_0^2 13 - y^2 \, dy - 2\pi \cdot 6 \int_0^2 \sqrt{4-y^2} \, dy$ → quarter of circle

$= 2\pi \left[13y - \frac{y^3}{3}\right]_0^2 - 12\pi \left[\frac{\pi(2)^2}{4}\right]$

$= 2\pi \left[26 - \frac{8}{3}\right] - 12\pi^2$

$= 2\pi \cdot \frac{70}{3} - 12\pi^2$

$= 2\pi \left(\frac{70}{3} - 6\pi\right)$ units^3

Question 13 (Miss Lee)

a) check the grid

b) $\int \sin^2 3x \, dx = \int \frac{1}{2}(1-\cos 6x)\, dx$

$= \frac{1}{2}\int 1 - \cos 6x \, dx$

$= \frac{1}{2}x - \frac{1}{12}\sin 6x + C$

c) i) $\cos 4\theta = \cos(3\theta + \theta)$

$= \cos 3\theta \cos\theta - \sin 3\theta \sin\theta$

$= (4\cos^3\theta - 3\cos\theta)\cos\theta - (3\sin\theta - 4\sin^3\theta)\sin\theta$

$= 4\cos^4\theta - 3\cos^2\theta - 3\sin^2\theta + 4\sin^4\theta$

$= 4(\cos^4\theta + \sin^4\theta) - 3(\cos^2\theta + \sin^2\theta)$

$\cos 4\theta = 4(\cos^4\theta + \sin^4\theta) - 3$

$4(\cos^4\theta + \sin^4\theta) = \cos 4\theta + 3$

$\therefore \cos^4\theta + \sin^4\theta = \frac{1}{4}(\cos 4\theta + 3)$

ii) If $x = \cos\theta$,

$8x^4 + 8(1-x^2)^2 = 7$

$8\cos^4\theta + 8(1-\cos^2\theta)^2 = 7$

$8\cos^4\theta + 8(\sin^2\theta)^2 = 7$

$8\cos^4\theta + 8\sin^4\theta = 7$

$\cos^4\theta + \sin^4\theta = \frac{7}{8}$

$\therefore \frac{1}{4}(\cos 4\theta + 3) = \frac{7}{8}$

$\cos 4\theta + 3 = \frac{7}{2}$

$\cos 4\theta = \frac{1}{2}$

$4\theta = \frac{\pi}{3}, \frac{5\pi}{3}, \frac{7\pi}{3}, \frac{11\pi}{3}, \frac{13\pi}{3}, \frac{17\pi}{3}, \frac{19\pi}{3}, \frac{23\pi}{3}$

$\therefore \theta = \frac{\pi}{12}, \frac{5\pi}{12}, \frac{7\pi}{12}, \frac{11\pi}{12}, \frac{13\pi}{12}, \frac{17\pi}{12}, \frac{19\pi}{12}, \frac{23\pi}{12}$

$\cos\frac{\pi}{12} = 0.9659\ldots$ $\cos\frac{5\pi}{12} = 0.2588\ldots$

$\cos\frac{7\pi}{12} = \cos(\pi - \frac{5\pi}{12}) = -\cos\frac{5\pi}{12} = -0.2588\ldots$

$\cos\frac{11\pi}{12} = \cos(\pi - \frac{\pi}{12}) = -\cos\frac{\pi}{12} = -0.9659\ldots$

$\therefore x = \cos\frac{\pi}{12}, \cos\frac{5\pi}{12}, -\cos\frac{5\pi}{12}, -\cos\frac{\pi}{12}$

iii) $8x^4 + 8(1-x^2)^2 = 7$

$8x^4 + 8(1 - 2x^2 + x^4) = 7$

$8x^4 + 8x^4 - 16x^2 + 8 - 7 = 0$

$16x^4 - 16x^2 + 1 = 0$

* product of roots;

$\cos\frac{\pi}{12} \times \cos\frac{5\pi}{12} \times -\cos\frac{\pi}{12} \times -\cos\frac{5\pi}{12} = \frac{e}{a} = \frac{1}{16}$

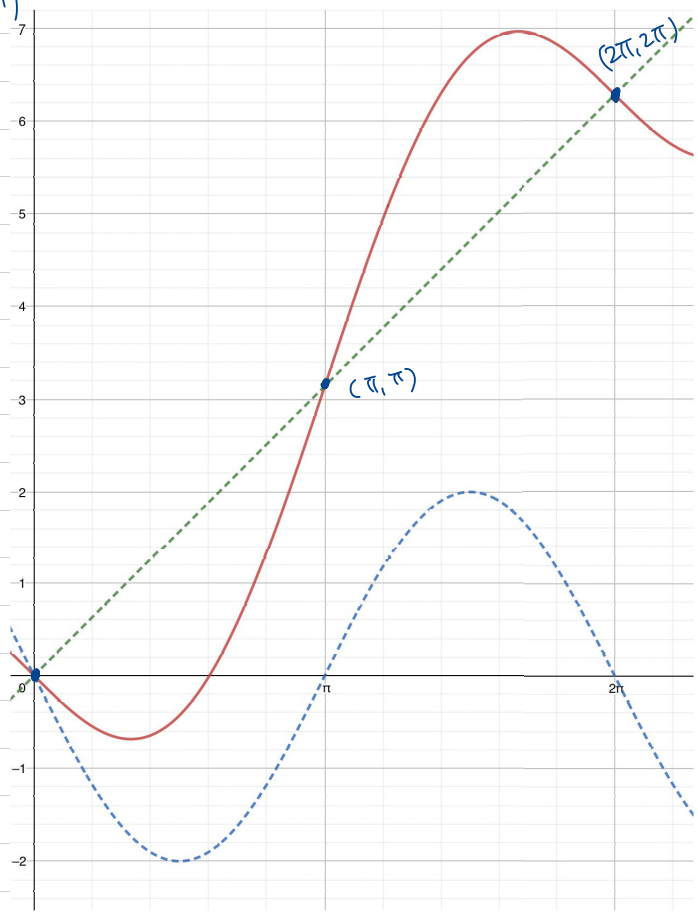
$(\cos\frac{\pi}{12})^2 (\cos\frac{5\pi}{12})^2 = \frac{1}{16}$

$\cos\frac{\pi}{12} \cdot \cos\frac{5\pi}{12} = \pm\frac{1}{4}$

$\because \cos\frac{\pi}{12} > 0$ & $\cos\frac{5\pi}{12} > 0$

$\cos\frac{\pi}{12} \cdot \cos\frac{5\pi}{12} = \frac{1}{4}$

a)



Question 14 (Mr Sekaran)

a) i) $\frac{dx}{dt} = x \sin t$

$\int \frac{1}{x}\, dx = \int \sin t\, dt$

$\ln |x| = -\cos t + C$

$x = e^{-\cos t + C}$

$\therefore x = Ae^{-\cos t}$

when $t = 0,\ x = 1$;

$1 = Ae^{-\cos 0}$

$1 = Ae^{-1}$

$\therefore A = e$

$\therefore x = e \cdot e^{-\cos t}$

$= e^{-\cos t + 1}$

ii) max x when $-\cos t + 1$ is max

$\therefore t = \pi$

b) i) $\frac{3g}{L} \times \frac{wL}{min} = 3w\ g/min$

ii) $\frac{Q}{1500}\ g/L$

iii) $\frac{Qg}{1500L} \times \frac{wL}{min} = \frac{Qwg}{1500\ min}$

iv) $\frac{dQ}{dt} = in - out$

$= 3w - \frac{Qw}{1500}$

$= \frac{w}{1500}(4500 - Q)$

v) $\frac{dQ}{dt} = \frac{w}{1500}(4500 - Q)$

$\frac{dQ}{dt} = -\frac{w}{1500}(Q - 4500)$

$\int \frac{1}{Q - 4500}\, dQ = \int -\frac{w}{1500}\, dt$

$\ln |Q - 4500| = -\frac{w}{1500} t + C$

$Q - 4500 = e^{-\frac{w}{1500} t + C}$

$Q = 4500 + Ae^{-\frac{wt}{1500}}$

vi) when $t = 0,\ Q = 0$;

$0 = 4500 + Ae^{0}$

$\therefore A = -4500$